[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1027 **D**

Unique Paper Code : 2352011101

Name of the Paper : DSC-1 : Algebra

Name of the Course : **B.Sc. (H) Mathematics, UGCF-2022**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Prove that one root of $x^3 + px^2 + qx + r = 0$ is negative of another root if and only if $r = pq$. (7.5)

   (b) Solve $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetical progression. (7.5)

(c) Find all the integral roots of

$$x^7 + 2x^5 + 4x^4 - 8x^2 - 32 = 0. \quad (7.5)$$

2. (a) Find the polar representation of the complex number

$$z = \text{Sin } a + i(1 + \text{Cos } a),\ a \in [0, 2\pi) \quad (7.5)$$

(b) Find |z| and arg z for $z = \dfrac{(2\sqrt{3}+2i)^8}{(1-i)^6} + \dfrac{(1+i)^6}{(2\sqrt{3}-2i)^8}$

(7.5)

(c) Find the geometric image for the complex number z such that

$$|z + 1 + i| < 3 \text{ and } 0 < \arg z < \frac{\pi}{6} \quad (7.5)$$

3. (a) Let $U_n = \{\epsilon_0, \epsilon_1, \epsilon_2, \ldots, \epsilon_{n-1}\}$ be the set of n^{th} roots of unity, where $\epsilon_k = \cos(2k\pi/n) + i\sin(2k\pi/n)$, $k \in \{0, 1, 2, \ldots, n-1\}$. Prove the following :

(i) Prove that $\epsilon_j . \epsilon_k \in U_n$ for all $j, k \in \{0, 1, 2, \ldots, n-1\}$

(ii) $\epsilon_j^{-1} \in U_n$ for all $j \in \{0, 1, 2, \ldots, n-1\}$

(4+3.5)

(b) Let a be an integer, show that there exists an integer k such that

$$a^2 = 3k \text{ or } a^2 = 3k + 1. \quad (7.5)$$

(c) (i) Prove that gcd(n, n+1) = 1 for every natural number n. Find integers x and y such that n.x + (n+1).y = 1.

(ii) Let a, b and c be three natural numbers such that gcd(a, c) = 1 and b divides c. Prove that gcd(a, b) = 1. (4+3.5)

4. (a) Let n > 1 be a fixed natural number. Let a, b, c be three integers such that ac ≡ bc (mod n) and gcd(c, n) = 1. Prove that a ≡ b (mod n).

(7.5)

(b) Solve the congruence, 7x ≡ 8 (mod 11). (7.5)

(c) Solve the following pair of congruences:

2x + 3y ≡ 1 (mod 6)

x + 3y ≡ 5 (mod 6) (7.5)

5. (a) Let G be the set of all 2×2 real matrices with non-zero determinant. Show that G is a group under the operation of matrix multiplication. Further show that it is not an Abelian Group. (7.5)

(b) Let G be a group such that for any x, y, z in the group, xy = zx implies y = z (called left-fight cancellation property). Show that G is Abelian. Give an Example of a non-abelian group in which left-right cancellation property does not hold. (7.5)

(c) Show that the set G = {1, 5, 7, 11} is a group under multiplication modulo 12 with the help of the Cayley table. (7.5)

6. (a) Show that for any integer n, the set $H_n = \{n.x \mid x \in Z\}$ is a subgroup of the group Z of integers under the operation of addition. Further show that $H_2 \cup H_3$ is not a subgroup of Z. (5.5+2)

(b) Let G be a group. Show that $|aba^{-1}| = |b|$ for all a and b in G (|x| denotes the order of an element x in G). (7.5)

(c) Show that the group $Z_n = \{0, 1, 2, ..., n-1\}$ is cyclic under the operation of addition modulo n. How many generators Z_n have? Further, describe all the subgroups of Z_{40}. (2+1+4.5)

(2)

[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1067 **C**

Unique Paper Code : 32351101

Name of the Paper : BMATH 101 C1 – Calculus

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the sections are compulsory.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

SECTION – I

Attempt any **four** *questions from Section – I.*

1. Sketch the graph of the function $f(x) = \frac{1}{3}x^3 - 9x + 2$ by finding intervals where it increases and decreases, relative extrema, concavity and inflection points (if any).

2. Evaluate the following limit

$$\lim_{x \to 0} \frac{\tan^{-1}(3x) - 3\tan^{-1} x}{x^3}$$

3. Determine whether the graph of following function has a vertical tangent or a cusp

$$f(x) — x^{2/3}(2x + 5)$$

4. It is projected that t years from now, the population of a certain country will be $P(t) = 50e^{0.02t}$ millions.

(a) At what rate will the population be changing with respect to time 10 years from now?

(b) At what percentage rate will the population be changing with respect to time t years from now?

5. If $y = e^{m\cos^{-1}x}$, show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$$

SECTION – II

Attempt any **three** *questions from Section – II.*

6. Sketch the graph of the curve $r = 3\sin 2\theta$ in polar coordinates.

7. Find an equation for a hyperbola that satisfies the condition that the curve has vertices $(\pm 1, 0)$ and asymptotes $y = \pm 2x$.

8. Describe the graph of the equation :

$$x^2 + 4y^2 + 6x - 40y + 93 = 0.$$

9. Identify and sketch the curve :

$$x^2 + 4xy - 2y^2 - 6 = 0.$$

SECTION – III

Attempt any **four** *questions from Section – III.*

10. Find the arc length of the parametric curve :
$x = \cos t$, $y = t + \sin t$ for $0 \le t \le \pi$.

11. Find the area of the surface generated by revolving the curve $x = \sqrt{25 - y^2}$, $-3 \le y \le 3$ about the y-axis.

12. The region bounded by the curves $y = x$ and $y = x^2$ is rotated about the line $y = 4$. Compute the volume of the resulting solid.

13. Find the value of the integral $\int_0^{\ln 5} \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) dx$.

14. Evaluate $\int \sin^2 x \cos^4 x \, dx$.

SECTION – IV

Attempt any **four** *questions from Section – IV.*

15. For what values of t, the vector function $F(t) = e^t \left[t\hat{i} + \frac{1}{t}\hat{j} + 3\hat{k} \right]$ is continuous?

16. Show that the vector valued function
$$R(t) = \left(2\hat{i} + 2\hat{j} + \hat{k}\right) + \left(\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right)\cos t + \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)\sin t$$
describes the motion of a particle moving in the circle of radius 1, centered at the point (2,2,1) and lying in the plane $x + y - 2z = 2$.

17. A shell fired from ground level at an angle of 45° hits the ground 2000 m away. What is the muzzle speed of the shell?

18. A baseball hit at a 24° angle from 3 ft above the ground just goes over the 9-ft fence 400 ft from home plate. About how fast was the ball traveling, and how long did it take the ball to reach the wall?

19. Find the curvature $\kappa(t)$ for the circular helix

$x = a\cos t$, $y = a\sin t$, $z = ct$, where a and c are constants and $a > 0$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1264 **D**

Unique Paper Code : 2354001001

Name of the Paper : GE: Fundamentals of Calculus

Name of the Course : **Common Prog. Group**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. This question paper has six questions.
4. Attempt any **two** parts from each question.

1. (a) (i) Establish that $\lim_{x \to 0} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$ does not exist.

(ii) Examine the continuity of the function

$$g(x) = \begin{cases} -x^2 & , \text{ if } x \le 0 \\ 5x - 4 & , \text{ if } 0 < x \le 1 \\ 4x^2 - 3x, & \text{ if } 1 < x < 2 \\ 3x + 4 & , \text{ if } x \ge 2 \end{cases}$$

at $x = 0, 1, 2$ and discuss their type of discontinuities, if any.

(b) Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$. Also prove that if $x^y = e^{x-y}$, then $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

(c) Find the n^{th} derivatives of $f(x) = e^{ax}\cos^2 bx$ and $g(x) = \sin^3 x \sin 3x$.

2 (a) If $y = e^{m\sin^{-1}x}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. Also find $y_n(0)$.

(b) Let $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ and $v = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$. Show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0 \text{ and } x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = \tan V$$

(c) If $V = r^m$ where $r^2 = x^2 + y^2 + z^2$, then prove that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1)r^{m-2}$$

3 (a) State and prove Rolle's theorem. Verify it for the function

$f(x) = x^3 - 6x^2 + 11x - 6$ in the domain [1,3].

(b) State Lagrange's mean value theorem. Use it to show that

$$\frac{x}{1+x} < \log(1+x) < x, \text{ for all } x > 0.$$

(c) Verify Cauchy's mean value theorem for the following pair of functions:

(i) $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x}$ in the domain [2,5]

(ii) $f(x) = \sin x$ and $g(x) = \cos x$ in the domain $[0, \pi/2]$

(iii) $f(x) = e^x$ and $g(x) = e^{-x}$ in the domain [1,4]

4 (a) Find the range of x for which the series $a + ax + ax^2 + \dots + ax^{n-1} + \dots$ is convergent, where a is a nonzero real number. Verify whether the series $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \cdots$ is convergent or not.

(b) Find the Taylor's series for $f(x) = \sin x$ and $g(x) = \cos x$.

(c) Evaluate the following:

(i) $\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$

(ii) $\lim_{x\to 0}\left(\frac{\tan^2 x - x^2}{x^2 \tan^2 x}\right)$

5. (a) Determine the intervals of concavity and points of inflection of the curve $y = 3x^5 - 40x^3 + 3x - 20$. Also use both first and second derivative tests to show that $f(x) = x^3 - 3x + 3$ has relative minimum at $x = 1$.

(b) Find asymptotes of the curve :
$y^3 - 2xy^2 - x^2y + 2x^3 + 2x^2 - 3xy + x - 2y + 1 = 0$.

(c) Determine the intervals of concavity and points of inflection of the curve $y = e^{-x^2}$. Also, show that the points of inflection of the curve $y = -(x-3)\sqrt{(x-5)}$ lies on the line $3x = 17$.

6. (a) Sketch a graph of $y = \frac{x}{x^2+4}$ and identify the locations of all asymptotes, intercepts, relative extrema and inflection points.

(b) Locate the critical points and identify which critical points are stationary points for the functions:

(i) $f(x) = 4x^4 - 16x^2 + 17$

(ii) $g(x) = 3x^4 + 12x$

(iii) $h(x) = 3x^{5/3} - 15x^{2/3}$

(c) Trace the curve $r = 2(1 + \cos\theta)$.

(2000)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1497 **C**

Unique Paper Code : 42351101

Name of the Paper : Mathematics – I (Calculus and Matrices)

Name of the Course : **B.Sc. (Math. Sci.) – I / B.Sc. (Phy. Sci.) – I / B.Sc. (Life Sci.) – I (CBCS (LOCF))**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** sections are compulsory.

3. Attempt any **two** questions from each section.

P.T.O.

SECTION I

1. (a) Let $f(x)=\begin{cases} x-2, & x<0, \\ x^2, & 0\le x\le 2, \\ 2x, & x>2. \end{cases}$

Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 2} f(x)$, if they exist.

(b) Find a value of the constant m, if possible, that will make the function continuous everywhere

$$f(x)=\begin{cases} 9-x^2, & x\ge -3, \\ m/x^2, & x<-3. \end{cases}$$

(c) Find $\frac{d^n y}{dx^n}$ where $y = \cos^4 x$. (5+5+5)

2. (a) Show that the function $f(x) = |x|$ is differentiable on $\mathbb{R}\setminus\{0\}$ but has no derivative at $x = 0$.

(b) If $y = \sin(m \sin^{-1} x)$ then show that

$$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n.$$

(c) Find the Maclaurin series for the function $f(x) = \cos 2x$ assuming the validity of expansion. (5+5+5)

3. (a) State and prove Rolle 's Theorem. Also discuss its geometrical interpretation.

(b) Find the value of ? for the following function that satisfies the hypothesis of Lagrange's mean value theorem

$$f(x)=x+\frac{1}{x}, \quad \left[\frac{1}{2},2\right].$$

Verify Legrange's mean value theorem for the function $f(x)=x+\frac{1}{x}$ in the interval $\left[\frac{1}{2},2\right]$.

(c) Draw the level curves for the surface

$f(x,y)=\sqrt{16x^2+25y^2}$ at heights

k = 1,2,5 and 7. (5+5+5)

SECTION II

4. (a) Row reduce the matrix A below to echelon form, and locate the pivot columns of A where

$$A=\begin{bmatrix}0 & -3 & -6 & 4 & 9\\ -1 & -2 & -1 & 3 & 1\\ -2 & -3 & 0 & 3 & -1\\ 1 & 4 & 5 & -9 & -7\end{bmatrix}$$

(b) While giving proper justification, find h and k so that the system

$$x + 3y = 2$$

$$3x + hy = k$$

has

(i) No solution

(ii) Unique solution

(iii) Infinite solution (6.5+6)

5. (a) Define horizontal shear linear transformation by a factor k on $\mathbb{R}^2$.

Find its standard matrix. Find effect of horizontal shear by factor 2 on a unit square with vertices (0,0), (1,0), (1,1), (0,1).

(b) Is set $S=\left\{\begin{bmatrix}1\\-1\\2\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\2\end{bmatrix}\right\}$ a linearly independent or linearly dependent set? Justify. (6.5+6)

6. (a) After diagonalising A, compute A^8 where

$$A=\begin{bmatrix}4 & -3\\ 2 & -1\end{bmatrix}.$$

(b) Solve, (if consistent) the system of linear equations

$$x - y + z = 5$$

$$2x + y + z = 4$$

$$2y + 3z = 7$$

if it is consistent. (6.5+6)

SECTION III

7. (a) Find all complex numbers z such that $|z| = 1$ and

$$\frac{z}{\bar{z}} + \frac{\bar{z}}{z} = 1.$$

(b) Let $z_1 = 1 - i$ and $z_2 = \sqrt{3} + i$. Find $z_1 z_2$.

(c) Find the polar representation of the complex number $\frac{1}{2} + i\frac{\sqrt{3}}{2}$. (3+3+4)

8. (a) Find the cube roots of the number $z = 1 + i$.

(b) Find the equation of the straight line joining the points whose affixes are

$1 - i$ and $2 - 5i$.

(c) If n is a positive integer, then show that

$$(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos\frac{n\pi}{4}.$$ (3+3+4)

9. (a) Find the modulus and the argument of the complex number

$$\frac{(1+\cos\theta + i\sin\theta)^5}{(\cos\theta + i\sin\theta)^3}$$

(b) Find the equation of the circle whose radius is 3 and whose centre has the affix $1 - i$.

(c) Solve the equation :

$$z^7 - 2i\, z^4 - i\, z^3 - 2 = 0. \qquad (3+3+4)$$

5/12/22

[This question paper contains 4 printed pages.]

Your Roll No...............

D

Sr. No. of Question Paper : 1046

Unique Paper Code : 2352011102

Name of the Paper : DSC-2: Elementary Real Analysis

Name of the Course : **B.Sc. (H) Mathematics (UGCF-2022)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **three** parts from each question.
3. **All** questions carry equal marks.

1. (a) If $a \in \mathbb{R}$ is such that $0 \leq a < \epsilon$ for any $\epsilon > 0$, then show that $a = 0$.

   (b) Find all values of x that satisfy $|x - 1| > |x + 1|$. Sketch the graph of this inequality.

   (c) Find the supremum and infimum, if they exist, of the following sets:

   (i) $\left\{\cos\frac{n\pi}{2} : n \in \mathbb{N}\right\}$

   (ii) $\left\{\frac{x+2}{3} : x > 3\right\}$

P.T.O.

(d) Show that $\text{Sup}\left\{1-\frac{1}{n} : n \in \mathbb{N}\right\}=1$.

2. (a) Let S be a non-empty subset of $\mathbb{R}$ that is bounded. Prove that

$$\text{Inf } S = -\text{Sup } \{-s : s \in S\}$$

(b) State and prove the Archimedean Property of real numbers.

(c) If $S=\left\{\frac{1}{n}-\frac{1}{m} : n, m \in \mathbb{N}\right\}$, find *Inf* S and Sup S.

(d) Define a convergent sequence. Show that the limit of a convergent sequence is unique.

3. (a) Using the definition of limit, show that

$$\lim_{n\to\infty}\frac{2n+3}{3n-7}=\frac{2}{3}$$

(b) Show that $\lim_{n\to\infty}\left(n^{1/n}\right)=1$.

(c) State and prove the Sandwich Theorem for sequences.

(d) Show that every increasing sequence which is bounded above is convergent.

4. (a) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2+x_n}$ for all $n \geq 1$. Prove that $\langle x_n \rangle$ converges and find its limit.

(b) Prove that every Cauchy sequence is bounded.

(c) Show that the sequence $\langle x_n \rangle$ where

$$x_n = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}, \text{ for all } n \in \mathbb{N},$$

does not converge.

(d) Find the limit superior and limit inferior of the following sequences:

(i) $x_n = (-2)^n\left(1+\frac{1}{n}\right)$, for all $n \in \mathbb{N}$

(ii) $x_n = (-1)^n\left(\frac{1}{n}\right)$, for all $n \in \mathbb{N}$

5. (a) Show that the geometric series $\sum_{k=1}^{\infty} ar^{k-1}$ converges if and only if $|r| < 1$.

(b) Find the sum of the following series, if it converges,

$$\sum\frac{1}{(n+a)(n+a+1)}, \; a>0$$

(c) Find the rational number which is the sum of the series represented by the repeating decimal $0.\overline{15}$.

(d) Check the convergence of the following series:

(i) $\sum\frac{1}{\log n}$, $n \geq 2$

(ii) $\sum\tan^{-1}\left(\frac{1}{n}\right)$

6. (a) State the Ratio Test (limit form) for positive series. Using this test or otherwise, check the convergence of the following series:

(i) $\sum\left(\frac{n!}{n^n}\right)$

(ii) $\sum\left(\frac{n!}{e^n}\right)$

(b) Check the convergence of the following series:

(i) $\sum_{n=2}^{\infty}\left(\frac{\log n}{n^2}\right)$

(ii) $\sum\left(\frac{n^{n^2}}{(n+1)^{n^2}}\right)$

(c) Define absolute convergence of a series. Show that every absolutely convergent series is convergent. Is the converse true? Justify your answer.

(d) Check the following series for absolute or conditional convergence :

(i) $\sum(-1)^{n+1}\left(\frac{n}{n^2+1}\right)$

(ii) $\sum_{n=2}^{\infty}(-1)^n\left(\frac{1}{n^2+(-1)^n}\right)$

Dec-2022

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1008 **D**

Unique Paper Code : 2352571101

Name of the Paper : Topics in Calculus

Name of the Course : **BA / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.

Unit I

1. (a) Use (ε, δ) definition of a limit to prove that $\lim_{x \to 3}(3x - 7) = 2$. (7.5)

P.T.O.

(b) Show that the function f(x) defined as $f(x) = |x-1| + |x+1|, x \in R$ is not derivable at the point x = −1 and x = 1, is derivable at every other point. (7.5)

(c) Discuss the continuity of f(x) = [x] at x = 1.

Also mention the type of discontinuity. (7.5)

2. (a) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, prove that

$$(1-x^2)y_{n+1} - (2n+1)xy_n - n^2 y_{n-1} = 0. \quad (7.5)$$

(b) (i) Find $\frac{d^2y}{dx^2}$, if $x = a(\cos\theta + \theta\sin\theta)$,

$y = a(\sin\theta - \theta\cos\theta)$.

(ii) If $z = x^y$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. (7.5)

(c) If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

(7.5)

Unit II

3. (a) State and prove Rolle's theorem. (7.5)

(b) What is the geometrical interpretation of Lagrange's mean value theorem. Find 'c' of Lagrange's mean value theorem for the following function:

$f(x) = x(x-1)(x-2)$; $a = 0$, $b = \frac{1}{2}$. (7.5)

(c) State the Cauchy mean value theorem. Find the value of 'c' of Cauchy's mean value theorem for the following functions

$f(x) = \sin x$ and $g(x) = \cos x$ in $[-\pi/2, 0]$ (7.5)

4. (a) State Taylor's theorem. Hence expand cosx in terms of $(x - \pi/4)$. (7.5)

(b) Assuming $R_n(x) \to 0$ as $n \to \infty$, find the Maclaurin's series expansion of $\sin(x/2)$. (7.5)

(c) (i) Evaluate $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$. (3)

(ii) For what value of a does $\frac{\sin 2x - a\sin x}{x^3}$ tend to a finite limit as $x \to 0$? (4.5)

Unit III

5. (a) Find all the asymptotes of the curve

$$2x^3 + 3x^2y - 3xy^2 - 2y^3 + 3x^2 - 3y^2 + y = 3. \quad (7.5)$$

(b) (i) Find the points of inflexion of the curve

$$y = 3x^4 - 4x^3 + 1. \quad (3)$$

(ii) Find the range of values of x in which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards and concave downwards. (4.5)

(c) Trace the curve $x^3 + y^3 = a^2x$. (7.5)

6. (a) Prove that at $x = a$ the curve $ay^2 = (x-a)^2(x-b)$ has a conjugate point if $a < b$, a node if $a > b$, a cusp if $a = b$. (7.5)

(b) Using the result

$$\int \sin^n x dx = -\frac{\cos x \sin^{n-1} x}{x} + \frac{n-1}{n}\int \sin^{n-2} x dx,$$

evaluate the definite integral $\int_0^{\pi/2} \sin^n x dx$, where n is a positive integer and $n \geq 2$. (7.5)

(c) Trace the curve $r = a \cos 3\theta$. (7.5)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1713 **C**

Unique Paper Code : 42354302

Name of the Paper : Algebra

Name of the Course : **B.Sc.(Prog)/Mathematical Sciences**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt **any two parts** from each question.
4. All questions are **compulsory**

Unit I

1. (a) Define the inverse of an element and show that inverse of an element in a group is unique. (6)

P.T.O.

(b) Let the element "α" belong to a group and $\alpha^{12} = e$. Express the inverse of each of the elements α, α^6, α^8 and α^{11} in the form α^k for some positive integer k. (6)

(c) Let $\sigma = (1,5,7)(2,5,3)(1,6)$. Then find σ^{99}. (6)

2. (a) Let G be a group and let H be a nonempty subset of G. If ab is in H whenever a and b are in H and a^{-1} is in H whenever a is in H, then H is a subgroup of G. (6)

(b) Let G be an Abelian group and H and K be subgroups of G. Then HK = {hk: h∈ H, k ∈ K} is a subgroup of G. (6)

(c) State and prove Lagrange's Theorem. Is the converse of this theorem true? (6)

3. (a) In a finite cyclic group, the order of an element divides the order of the group. (6)

(b) Find the inverse of $\left\{\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\right\}$ in $(2, Z_{11})$ (6)

(c) Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles. (6)

Unit II

4. (a) State the subring test. Check whether the set $\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in Z\right\}$ is a subring of the ring of all 2 × 2 matrices over Z, the set of integers. (6.5)

(b) Define a field. Prove that a finite commutative ring with unity having no zero divisors is a field. (6.5)

(c) Show that the set $[Q\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in Q\}$ forms a ring. Is it a field? If yes, justify your answer. (6.5)

Unit III

5. (a) Prove that intersection of two subspaces of a vector space V(F) is a subspace of V(F). Is the result true for the union of two subspaces? Justify your answer. (6.5)

(b) Define basis of a vector space over a field F. Prove that every element of a vector space uniquely expressible as a linear combination of elements of the basis. (6.5)

(c) Check whether the vectors (1,-1, 2), (-1,2,-4), (-1,-1,2) form a basis of R^3. (6.5)

6. (a) Let T: V->U be a Linear Transformation. Define null space N(T) and range R(T) of T Show that N(T) and R(T) are subspaces of V and U respectively. (6.5)

(b) Define Linear Transformation. Prove that there exists a Linear Transformation T: R^2 -> R^3 such that T (1,1) = (1,0,2) and T(2,3) = (1,-1,4). Find T (8, 11). (6.5)

(c) Let T: R^3-> R^3 be a Linear Transformation defined by T (x, y, z) = (x-y+2z, 2x+y, -x- 2y+2z). Find the Range, Rank, Kernal and Nullity of T. Verify the Dimension Theorem. (6.5)

[This question paper contains 6 printed pages.]

Your Roll No...............

C

Sr. No. of Question Paper : 1963

Unique Paper Code : 32355301

Name of the Paper : GE – III Differential Equations

Name of the Course : **Generic Elective / Other than B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Solve the following differential equations : (6.5)

(i) $\frac{dy}{dx} = y\tan x + x^2\cos x$

(ii) $(x^2 - ay)dx + (y^2 - ax)dy = 0.$

P.T.O.

(b) Solve the Initial Value problem.

$$x\frac{dy}{dx}-3y=x^5y^{1/3}, \quad y(0)=1.$$

Or

Solve the following differential equations :

$$\sin y\frac{dy}{dx}-2\cos x\cos y=-\cos x\sin^2 x. \qquad (6.5)$$

(c) Find a family of oblique trajectories that intersect the family of parabolas $y^2 = cx$ at angle 60 degrees. (6.5)

2. (a) Consider the differential equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0.$$

Show that x^2 and $1/x^2$ are linearly independent solutions of this equation on the interval $0 < x < \infty$. Write the general solution of the given equation. Hence find the particular solution satisfying the initial conditions $y(2) = 3$, $\frac{dy}{dx}(2)=-1$. (6)

(b) Find the suitable integrating factor for the differential equation

$(x^2 + y^2 + x)dx + xydy = 0$ and hence solve it. (6)

(c) Given that $y = e^{2x}$ is a solution of the differential equation

$$(2x-1)\frac{d^2y}{dx^2}-4(x+1)\frac{dy}{dx}+4y=0,$$

find a linearly independent solution by reducing the order. Write the general solution also. (6)

3. (a) Solve the Initial value Problem

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+5y=0, \quad y(0)=1, \quad \frac{dy}{dx}(0)=5. \qquad (6.5)$$

(b) Find the general solution of the differential equation using method of Undetermined Coefficients

$$\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}-3\frac{dy}{dx}-10y=8\left(xe^{-2x}\right) \qquad (6.5)$$

(c) Find the general solution of the differential equation using method of Variation of Parameters

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\frac{1}{1+e^x}.$$ (6.5)

4. (a) Given that $e^x \sin 2x$ is a solution of the differential equation

$$\frac{d^4y}{dx^4}+3\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+13\frac{dy}{dx}+30y=0$$

find the general solution. (6)

(b) Find the general solution of the differential equation by assuming $x > 0$

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=4\sin(\log x)$$ (6)

(c) Find the general solution of the given linear system

$$\frac{dx}{dt}=2x+5y, \quad \frac{dy}{dt}=5x+12.5y$$ (6)

5. (a) Find the solution to the linear partial differential equation

$$x u_x + y u_y = u + 1, \; u(x,y) = x^2 \text{ on } y = x^2.$$ (6.5)

(b) Find the solution of the following partial differential equation by the method of separation of variables

$$u_x + 2u_y = 0, \quad u(0,y) = 3\exp(-2y).$$ (6.5)

(c) Reduce the equation

$$u_x - y u_y - u = 1,$$

into canonical form and obtain the general solution. (6.5)

6. (a) Find the general solution of the differential equation by reducing it into the canonical form

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$$ (6)

(b) Reduce the equation

$$u_{xx} + 4u_{xy} + 4u_{yy} = 0$$

into canonical form and hence find its general solution. (6)

(c) (i) Find the partial differential equation by eliminating the arbitrary function f and g from the following equation

$$z = f(x + ay) + g(x - ay).$$

(ii) Find the partial differential equation by eliminating the arbitrary function f from the following equation

$$yz + zx + xy = f\left(\frac{z}{x + y}\right) \qquad (6)$$

3

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1409 **C**

Unique Paper Code : 32351302

Name of the Paper : BMATH306 – Group Theory-I

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question from **Q2** to **Q6**.
4. In the question paper, given notations have their usual meaning unless until stated otherwise.

P.T.O.

1. Give short answers to the following questions. Attempt any **six**.

   (i) What is the total no of rotations and total no of reflections in the dihedral group D_3? Describe them (rotations and reflections) in pictures or words. What can you say about the total no of rotations and total no of reflections in the dihedral group D_n?

   (ii) Give one non- trivial, proper subgroup of GL(2, R). Is GL(2, R) a group under addition of matrices? Answer in few lines.

   (iii) Let G be a group with the property that for any a, b, c in G,

   ab = ca implies b = c. Prove that G is Abelian.

   (iv) Give an example of a cyclic group of order 5. Show that a group of order 5 is cyclic.

   (v) Prove that a cyclic group is Abelian. Is the converse true?

   (vi) Find all subgroups of Z_{15}.

   (vii) Prove that 1 and -1 are the only two generators of (Z,+). Give short answer in few lines.

   (viii) "Z_n, $n \in N$, is always cyclic whereas U(n), $n \in N$; $n \geq 2$ may or may not be cyclic". Prove or disprove the statement in a few lines. (6×2=12)

2. (a) Let $G = \{a + b\sqrt{2} \mid$ a and b are rational nos not both zero$\}$

   Prove that G is a group under ordinary multiplication. Is it Abelian or Non-Abelian? Justify your answer.

(b) Prove that a group of composite order has a non-trivial, proper subgroup.

(c) Prove that order of a cyclic group is equal to the order of its generator. (2×6.5=13)

3. (a) Prove that every permutation of a finite set can be written as a cycle or as a product of distinct cycles. (6)

(b) (i) In S_4, write a cyclic subgroup of order 4 and a non-cyclic subgroup of order 4.

(ii) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 8 & 6 & 7 & 5 & 1 & 3 \end{bmatrix}$ and

$$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 5 & 1 & 8 & 3 & 2 & 6 \end{bmatrix}$$

Write α, β and $\alpha\beta$ as product of 2-cycles. (3+3=6)

(c) (i) Let $|a| = 24$. How many left cosets of $H = \langle a^4 \rangle$ in $G = \langle a \rangle$ are there? Write each of them.

(ii) State Fermat's Little theorem. Also compute 5^{25} mod 7 and 11^{17} mod 7. (3+3=6)

4. (a) (i) Let H and K be two subgroups of a finite group. Prove that

$HK \leq G$ if G is Abelian.

(ii) Give an example of a group G and its two subgroups H and K ($H \neq K$) such that HK is not a subgroup of G. (3+3.5=6.5)

(b) (i) Let G be a group and let Z (G) be the centre of G. If G/Z (G) is cyclic, prove that G is Abelian.

(ii) Let $|G| = pq$, p and q are primes. Prove that $|Z(G)| = 1$ or pq. (4+2.5=6.5)

(c) (i) Prove that a subgroup of index 2 is normal.

(ii) Let $G = U(32)$, $H = U_8(32)$. Write all the elements of the factor group G/H. Also find order of 3H in G/H. (3+3.5=6.5)

5. (a) Show that the mapping from $\mathbb{R}$ under addition to $GL(2, \mathbb{R})$ that takes x to $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ is a group homomorphism. Also, find the kernel of the homomorphism.

(b) Let ϕ be a homomorphism from a group G to a group $\bar{G}$. Show that if $\bar{K}$ is a subgroup of $\bar{G}$,

then $\phi^{-1}(\bar{K}) = \{k \in G: \phi(k) \in \bar{K}\}$ is a subgroup of G.

(c) If H and K are two normal subgroups of a group G such that $H \subseteq X$, then prove that

$$G/K \approx \frac{G/H}{K/H}$$ (2×6=12)

6. (a) Show that the mapping ϕ from $\mathbb{C}^*$ to $\mathbb{C}^*$ given by $\phi(z) = z^4$ is a homomorphism. Also find the set of all the elements that are mapped to 2.

(b) Prove that every group is isomorphic to a group of permutations.

(c) Let G be the group of non-zero complex numbers under multiplication and N be the set of complex numbers of absolute value 1.

Show that G/N is isomorphic to the group of all the positive real numbers under multiplication.

(2×6.5=13)

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1341 **C**

Unique Paper Code : 32353301

Name of the Paper : SEC: LaTeX and HTML

Name of the Course : **CBCS B.Sc.(H) Mathematics**

Semester : III

Duration : 2 Hours Maximum Marks : 38

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.

1. Fill in the blanks : (4×½=2)

   (i) _______ command is used to create diagonal dots in LaTeX.

   (ii) To use PSTricks, which package is included in the preamble _______.

   (iii) _______ tag is used for writing a paragraph of any text in the HTML.

P.T.O.

(iv) The ______ beamer command is used to show the elements of a list one point at a time.

2. Attempt any **eight** part : (8×2=16)

(i) Correct the following input as per LaTeX commands and write output also `$x = α and $y = β $ then $ frac{α}{β} = 2 $.`

(ii) Write codes in HTML to create a web page consisting of a link to a photograph. Keep the background color of the page light grey.

(iii) The input command in LaTeX to produce the following :

$$f(x)=\begin{cases}\dfrac{|x-a|}{x-a}, & \text{for } x \neq a\\ 0, & \text{for } x = a\end{cases}$$

(iv) Write the LaTeX commands to draw triangle and circles using the picture environment with at least one example of each type.

(v) Write the command to plot the function $x^{-5} + x^{5}$ in a PS Plot environment.

(vi) Define

(a) URL

(b) Hyperlink

(vii) Write the following to postfix expressions

(a) $\dfrac{x^2+5}{x^2-7}$

(b) $1+\sqrt[3]{x-1}$

(viii) Write the output of the following LaTeX command

```
\begin{eqnarray*}
e^x & = & \frac{x^0}{0!}+\frac{x^1}{1!}
+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\\
e^{-1} & = & \frac{{(-1)}^0}{0!}+\frac{{(-1)}^1}{1!}
+\frac{{(-1)}^2}{2!}+\frac{{(-1)}^3}{3!}+\cdots\\
& = & \frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}
-\frac{1}{3!}+\cdots
\end{eqnarray*}
```

(ix) Write the commands using PSTricks to draw a picture of a circle with a shaded sector of radius 5 cm.

(x) Write the commands using PSTricks to draw a dashed graph of $f(x)=\sqrt{x}$ in the interval [0,1].

3. Attempt any **four** parts : (4×5=20)

(i) Create a presentation with the following slides using the Beamer :

Slide 1: Title- Group; Author- ABC; Institute: XYZ University

Slide 2: Frame title- Definition of Group

A non-empty set G with binary operation · is said to be Group if for a, b ∈ G, we have

1. $a \cdot b \in G$
2. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
3. $\exists\, e \in G$ such that $a \cdot e = e \cdot a = a$
4. for every $a \in G, \exists\, a' \in G$ such that $a \cdot a' = a' \cdot a = e$.

Slide 3: Frame title- Examples

- $GL(2,R) = \left\{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} : a_{11}, a_{12}, a_{21}, a_{22} \in R \text{ and } a_{11}a_{22} - a_{12}a_{21} \neq 0\right\}$
- $(Z_n, \oplus_n)$, where $Z_n = \{0,1,2,\cdots,n-1\}$ and $\oplus_n$ is addition modulo n.

Slide 4: Thank You.

(ii) Write a code in LaTeX to typeset the following :

Let σ be a smooth parametric surface. Then the surface integral of f(x, y, z) over σ (independent of subdivision) is

$$\iint_{\sigma} f(x,y,z)\, dS = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*, y_k^*, z_k^*)\, \Delta S_k$$

where $(x_k^*, y_k^*, z_k^*) \in \sigma$ provided this limit exists.

(iii) Find the errors and write the corrected LaTeX source code (highlight your corrections using underline in the answer). Also, write its output.

```
\documentclass{article}
\title{Limits of x^n}
\author{ABC}
\begin{document}
\titlepage
$f(x)=x^n$ for n=1, 2, \cdots at $\pm \infinity$.
\begin{enumeration}
\item $\lim_{x\rightarrow +\infinity} = +\infinity $ for every $n$.
\item $\lim_{x\rightarrow -\infinity} x^n = \left\{\begin{array}{cc} -\infinity & n is odd\\
+\infinity & n is even \end{array}\right.
\end{document}
```

(iv) Draw the graph of the function $y = \sqrt{x}\left(\sin\frac{1}{x}\right)^2$ for $0 < x \leq 2$ using PS Tricks by labelling the graph and axes.

(v) Make two HTML webpages with the first page having hyperlink:

http://maths.du.ac.in/courses/BABSc/BABSc.html displays as

"Department of Mathematics" and the second page as below :

Department of Mathematics

University of Delhi

1. B.A./B.Sc. syllabus
    - B.Sc. (Hons) Mathematics
    - Generic Papers
2. M.Sc. syllabus
    - o Functional Analysis
    - o Field Theory

[This question paper contains 4 printed pages.]

Your Roll No...............

C

Sr. No. of Question Paper : 1427

Unique Paper Code : 32351303

Name of the Paper : BMATH 307 – Multivariate Calculus

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory
3. Attempt any **Five** questions from each section. **All** questions carry equal marks

SECTION 1

1. Let $f(x,y)=\dfrac{xy\left(x^2-y^2\right)x}{x^2+y^2}$ if $(x, y) \neq (0,0)$

$= 0$ otherwise

Show that $f(0, y) = -y$ and $f(x, 0) = x$ for all x and y.

P.T.O.

2. Use incremental approximation to estimate the function $f(x, y) = \sin(xy)$ at the point

$$\left(\sqrt{\frac{\pi}{2}}+.01, \sqrt{\frac{\pi}{2}}-.01\right)$$

3. If $z = xy + f(x^2 + y^2)$, show that $y\,\partial z/\partial x - x\,\partial z/\partial y = y^2 - x^2$.

4. Assume that maximum directional derivative of f at $P_0(1,2)$ is equal to 50 and is attained in the direction towards Q(3, −4). Find ∇f at $P_0(1, 2)$.

5. Find the absolute extrema of $f(x, y) = 2x^2 - y^2$ on the disk $x^2 + y^2 \le 1$.

6. Use Lagrange multiplier to find the distance from (0, 0, 0) to plane $Ax + By + Cz = D$ where at least one of A, B, C is nonzero.

SECTION II

1. Compute the integral $\int_0^1 \int_x^{2x} e^{y-x} dy\, dx$ with the order of integration reversed.

2. Use Polar double integral to show that a sphere of radius α has volume $\frac{4}{3}\pi a^3$.

3. Compute the area of region D bounded above by line $y = x$, and below by circle $x^2 + y^2 - 2y = 0$.

4. Find the volume of the solid bounded above by paraboloid $z = 6 - x^2 - y^2$ and below by $z = 2x^2 + y^2$.

5. Evaluate $\iiint_D \frac{dx\, dy\, dz}{\sqrt{x^2 + y^2 + z^2}}$, where D is the solid sphere $x^2 + y^2 + z^2 \le 3$.

6. Use a suitable change of variables to find the area of region R bounded by the hyperbolas xy=1 and xy=4 and the lines y=x and y=4x.

SECTION III

1. Find the mass of a wire in the shape of curve C: $x = 3\sin t$, $y = 3\cos t$, $z = 2t$ for $0 \le t \le \pi$ and density at point (x, y, z) on the curve is $\delta(x, y, z) = x$.

2. Find the work done by force

$$\vec{F}(x,y,z) = (y^2 - z^2)\hat{i} + (2yz)\hat{j} - (x^2)\hat{k}$$

on an object moving along the curve C given by $x(t) = t$, $y(t) = t^2$, $z(t) = t^3$, $0 \le t \le 1$.

3. Use Green's theorem to find the work done by the force field

P.T.O.

$$\vec{F}(x,y)=(3y-4x)\hat{i}+(4x-y)\hat{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

4. Use Stoke's theorem to evaluate the surface integral

$$\iint_S \left(\text{curl } \vec{F}.N\right) dS$$

where $F = x\,i + y^2 j + z\,e^{xy}\,k$ and S is that part of surface $z = 1 - x^2 - 2y^2$ with $z \geq 0$.

5. Use divergence theorem to evaluate the integral

$\iint_S \vec{F}.N\ dS$ where $\vec{F}(x,y,z)=(\cos yz)\hat{i}+e^{xz}\hat{j}+3z^2\hat{k}$,

where S is hemisphere surface $z=\sqrt{4-x^2-y^2}$ together with the disk $x^2 + y^2 \leq 4$, in x-yplane.

6. Evaluate the line integral $\int_C \vec{F}.d\vec{R}$

Where $\vec{F}(x, y) = [(2x - x^2y)\,e^{-xy} + \tan^{-1} y]\,\hat{i}$ + $\left[\dfrac{x}{y^2+1} - x^3 e^{-xy}\right]\hat{j}$ and C is the ellipse $9x^2 + 4y^2 = 36$.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1381 **C**

Unique Paper Code : 32351301

Name of the Paper : BMATH 305 – Theory of Real Functions

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Let $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A and $f: A \to \mathbb{R}$, then define limit of function f at c.

   Use $\varepsilon - \delta$ definition to show that $\lim_{x \to 1} \frac{x}{x+1} = \frac{1}{2}$.

   (6)

P.T.O.

(b) Let $f: A \to \mathbb{R}$, $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A. Then show that $\lim_{x \to c} f(x) = L$ if and only if for every sequence (x_n) in A that converges to c such that $x_n \neq c$, $\forall n \in \mathbb{R}$, the sequence $(f(x_n))$ converges to L. (6)

(c) Show that $\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right)$ does not exist in $\mathbb{R}$ but $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x^2}\right) = 0$. (6)

2. (a) Let $A \subseteq \mathbb{R}$, $f: A \to \mathbb{R}$, $g: A \to \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A. Show that if f is bounded on a neighborhood of c and $\lim_{x \to c} g(x) = 0$, then $\lim_{x \to c} (fg)(x) = 0$. (6)

(b) Let $f(x) = e^{1/x}$ for $x \neq 0$, then find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0^-} f(x)$. (6)

(c) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 2x & \text{if } x \text{ is rational} \\ x+3 & \text{if } x \text{ is irrational} \end{cases}$$

Find all the points at which f is continuous. (6)

3. (a) Let $A \subseteq \mathbb{R}$ and let f and g be real valued functions on A. Show that if f and g are continuous on A then their product fg is continuous on A. Also, give examples of two functions f and g such that both are discontinuous at a point $c \in A$ but their product is continuous at c. (7½)

(b) State and prove Boundedness Theorem for continuous functions on a closed and bounded interval. (7½)

(c) State Maximum-Minimum Theorem. Let $I = [a,b]$ and $f: I \to \mathbb{R}$ be a continuous function such that $f(x) > 0$ for each x in I. Prove that there exists a number $\alpha > 0$ such that $f(x) \geq \alpha$ for all x in I. (7½)

4. (a) Let $A \subseteq \mathbb{R}$ and $f: A \to \mathbb{R}$ such that $f(x) \geq 0$ for all $x \in A$. Show that if f is continuous at $c \in A$, then $\sqrt{f}$ is continuous at c. (6)

(b) Show that every uniformly continuous function on $A \subseteq \mathbb{R}$ is continuous on A. Is the converse true? Justify your answer. (6)

(c) Show that the function $f(x) = \frac{1}{x^2}$, $x \neq 0$ is uniformly continuous on $[a, \infty)$, for $a > 0$ but not uniformly continuous on $(0, \infty)$. (6)

5. (a) Let $I \subseteq \mathbb{R}$ be an interval, let $c \in I$, and let $f: I \to \mathbb{R}$ and $g: I \to \mathbb{R}$ be functions that are differentiable at c. Prove that if $g(c) \neq 0$, the function f/g is differentiable at n, and

$$\left(\frac{f}{g}\right)'(c) = \frac{f'(c)g(c) - f(c)g'(c)}{(g(c))^2}. \quad (6)$$

(b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = |x| + |x + 1|$, $x \in \mathbb{R}$. Is f differentiable everywhere in $\mathbb{R}$? Find the derivative of f at the points where it is differentiable. (6)

(c) State Mean Value Theorem. If $f: [a, b] \to \mathbb{R}$ satisfies the conditions of Mean Value Theorem and $f'(x) = 0$ for all $x \in (a,b)$. Then prove that f is constant on [a, b]. (6)

6. (a) Let I be an open interval and let $f: I \to \mathbb{R}$ have a second derivative on I. Then show that f is a convex function on I if and only if $f''(x) \geq 0$ for all $x \in I$. (6)

(b) Find the points of relative extrema of the functions $f(x) = |x^2 - 1|$, for $-4 \leq x \leq 4$. (6)

(c) Use Taylor's Theorem with n = 2 to approximate $\sqrt[3]{1+x}$, , $x > -1$. (6)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1132 **C**

Unique Paper Code : 32357501

Name of the Paper : DSE-I Numerical Analysis (LOCF)

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All **six** questions are compulsory.

3. Attempt any two parts from each question.

4. Use of non-programmable scientific calculator is allowed.

1. (a) Define fixed point of a function and construct an algorithm to implement the fixed point iteration scheme to find a fixed point of a function. Find the fixed point of $f(x) = 2x(1 - x)$. (6)

(b) Perform four iterations of Newton's Raphson method to find the positive square root of 18. Take initial approximation $x_0=4$. (6)

(c) Find the root of the equation $x^3 - 2x - 6 = 0$ in the interval (2, 3) by the method of false position. Perform three iterations. (6)

2. (a) Define the order of convergence of an iterative method for finding an approximation to the root of $g(x) = 0$. Find the order of convergence of Newton's iterative formula. (6.5)

(b) Find a root of the equation $x^3 - 4x - 8 = 0$ in the interval (2, 3) using the Bisection method till fourth iteration. (6.5)

(c) Perform three iterations of secant method to determine the location of the approximate root of the equation $x^3 + x^2 - 3x - 3 = 0$ on the interval (1, 2). Given the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the approximations just obtained. (6.5)

3. (a) Using scaled partial pivoting during the factor step, find matrices L, U and P such that LU = PA

where $A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{pmatrix}$ (6.5)

(b) Set up the SOR method with w=0.7 to solve the system of equations:

$$3x_1 - x_2 + x_3 = 4$$

$$2x_1 - 6x_2 + 3x_3 = -13$$

$$-9x_1 + 7x_2 - 20x_3 = 7$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Jacobi iteration scheme to solve the system of equations:

$$10x_1 + x_2 + 4x_3 = 31$$

$$x_1 + 10x_2 - 5x_3 = -23$$

$$3x_1 - 2x_2 + 10x_3 = 38$$

Take the initial approximation as $X^{(0)} = (1, 1, 0)$ and do three iterations. (6.5)

4. (a) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data:

x	1	2	4	8
f(x)	3	7	21	73

(6)

(b) Calculate the Newton second order divided difference $\frac{1}{x^2}$ of based on the points x_0, x_1, x_2. (6)

(c) Obtain the Lagrange form of the interpolating polynomial for the following data:

x	1	2	5
f(x)	-11	-23	1

(6)

5. (a) Find the highest degree of the polynomial for which the second order backward difference approximation for the first derivative

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}$$

provides the exact value of the derivative irrespective of h. (6)

(b) Derive second-order forward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)}{2h}.$$

(6)

(c) Approximate the derivative of $f(x) = \sin x$ at $x_0 = \pi$ using the second order central difference formula taking h = ½, ¼ and 1/8 and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the Simpson's rule, approximate the value of the integral $\int_2^5 \ln x \, dx$. Verify that the theoretical error bound holds. (6.5)

(b) Apply Euler's method to approximate the solution of initial value problem $\frac{dx}{dt} = \frac{e^t}{x}$, $0 \le t \le 2$, $x(0) = 1$ and N = 4.

Given that the exact solution is $x(t) = \sqrt{2e^t - 1}$, compute the absolute error at each step. (6.5)

(c) Apply the optimal RK2 method to approximate the solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $1 \le t \le 2, x(1) = 1$ taking the step size as h = 0.5. (6.5)

[This question paper contains 8 printed pages.]

Your Roll No...............

C

Sr. No. of Question Paper : 1231

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V (under CBCS (LOCF) Scheme)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the given **eight** questions are compulsory to attempt.
3. Do any **two** parts from each of the given **eight** questions.
4. Marks for each part are indicated on the right in brackets.

P.T.O.

SECTION I

1. (a) Let N_0 be the set of non-negative integers. Define a relation $\leq$ on N_0 as: For m, $n \in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \leq N_0$: $n = km$. Then show that $\leq$ is an order relation on N_0. (2½)

(b) If '1', '2', '3' denote chains of one, two, three elements respectively and $\overline{3}$ denotes anti chain of three elements, then draw the Hasse diagram for the dual of $L \oplus K$ when $L = \overline{3}$ and $K = 1 \oplus (2 \times 2)$. (2½)

(c) Define maximum and a maximal element of a partially ordered set P. Give an example each for both definitions. (2½)

2. (a) Let P and Q be finite ordered sets and let $\psi: P \rightarrow Q$ be a bijective map. Then show that the following are equivalent :

(i) $x < y$ in P iff $\psi(x) < \psi(y)$ in Q

(ii) $x \prec y$ in P iff $\psi(x) \prec \psi(y)$ in Q (3)

(b) Define upper bound and lower bound of a subset S of a partially ordered set P. Construct an example of a partially ordered set P and its subset S and give the set of all upper bounds and lower bounds of S. (3)

(c) Let P and Q be ordered sets. Then show that the ordered sets P and Q are order isomorphic iff there exist order preserving maps $\varphi: P \rightarrow Q$ and $\psi: Q \rightarrow P$ such that :

$\varphi o \psi = id_Q$ and $\psi o \varphi = id_P$ where $id_S : S \rightarrow S$ denotes the identity map on S given by: $id_S(x) = x$, $\forall x \in S$. (3)

SECTION II

3. (a) Let $D_{60} = \{1, 2, 4, 5, 6, 12, 20, 30, 60\}$ be an ordered subset of $N_0 = N \cup \{0\}$, N being the set of natural numbers. If '$\leq$' is defined on D_{60} by $m \leq n$ if and only if m divides n then show that D_{60} does not form a lattice. Also Draw the diagram of D_{60} and find elements a, b, c, d $\in D_{60}$ such that $a \vee b$ and $c \wedge d$ do not exist in D_{60}. (5½)

(b) Define sublattice of a lattice. Prove that every chain of a lattice L is a lattice and also a sublattice of L. (5½)

(c) Define modular lattice. Prove that a homomorphic image of modular lattice is modular. (5½)

4. (a) Let L be a lattice. For any a, b, c ∈ L, show that the following inequalities hold :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $a \geq c \Rightarrow a \wedge (b \vee c) \geq (a \wedge b) \vee c$

(5)

(b) Let $(L, \wedge, \vee)$ be an algebraic lattice. If we define

$$a \leq b : \Leftrightarrow a \vee b = b$$

then show that $(L, \leq)$ is a lattice ordered set. (5)

(c) Let L_1 and L_2 be distributive lattices. Prove that the product $L_1 \times L_2$ is a distributive lattice. (5)

SECTION III

5. (a) A voting-machine for three voters has YES-NO switches. Current is in the circuit precisely when YES has a majority. Draw the corresponding contact diagram and the switching/circuit diagram. (5½)

(b) Show that a Boolean Algebra is relatively complemented. (5½)

(c) Simplify the polynomial :

$$f = x'yz + x'yz' + x'y'z + xy'z' + xy'z$$

using Quine's McCluskey method. (5½)

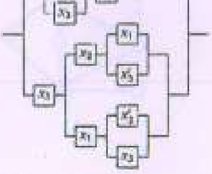
6. (a) Define a system of normal forms. Find conjunctive normal form for $p = y'z' + x'yz$. (5)

(b) Simplify the Boolean expression :

$$f = w'xy'z + w'xyz + w'xyz' + wxy'z + wxyz + wxyz' + wx'y'z + wx'yz$$

using Karnaugh Diagram. (5)

(c) Find the symbolic gate representation of the contact diagram : (5)



SECTION IV

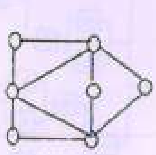
7. (a) (i) Show that the sum of the degrees of the vertices of a pseudograph is an even number equal to twice the number of edges.

(ii) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have? (5½)

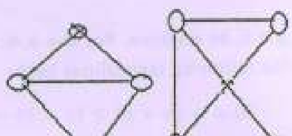
(b) (i) Define the degree sequence of a graph. Does there exist a graph with following degree sequence 6, 6, 5, 5, 4, 4, 4, 4, 3?

(ii) Show that the number of vertices of odd in a graph must be even. (5½)

(c) (i) What is a bipartite graph? Determine whether the graph given below is bipartite or not. Give the bipartition sets or explain why the graph is not bipartite.

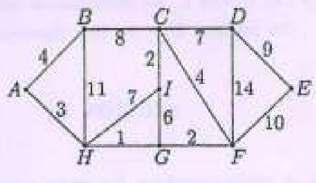


(ii) Define isomorphism of graphs. Also label the following graphs so as to show an isomorphism. (5½)



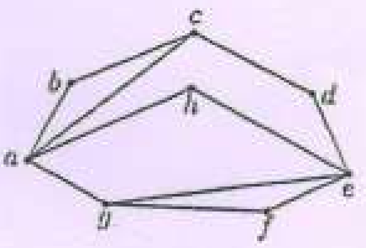
8. (a) Construct a Gray Code of length 3 using the concept of Hamiltonian Cycles. (5½)

(b) Apply Dijkstra's algorithm to find a shortest path from A to all other vertices in the weighted graph shown. (5½)



(c) (i) Does there exist a graph G with 28 edges and 12 vertices each of degree 3 or 6?

(ii) Define Eulerian circuit. Is the given graph Eulerian? *Give* reasons for your answer.



(5½)

5/12/22

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1049 **C**

Unique Paper Code : 32351502

Name of the Paper : BMATH512: Group Theory-II

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Question No. **1** has been divided into **10** parts and each part is of **1.5** marks.
4. Each question from Q. Nos. **2** to **6** has **3** parts and each part is of **6** marks. Attempt any **two** parts from each Question.

1. State true (T) or false (F). Justify your answer in brief.

   (i) A group of order p^2, p is a prime is always isomorphic to Z_{p^2}.

(ii) A group of order 15 is always cyclic.

(iii) A group of order 14 is simple.

(iv) The smallest positive integer n such that there are two non-isomorphic groups of order n is 6.

(v) Every inner automorphism induced by an element 'a' of group G is an automorphism of G.

(vi) A abelian group of order 12 must have an element of order either 2 or 3.

(vii) U(105) is isomorphic to external direct product of U(21) and U(5).

(viii) Center of a group G is always a subgroup of normalizer of A in G, where A is any subset of G.

(ix) $Aut(Z_{10})$ is a cyclic group of automorphisms of G.

(x) The largest possible order for an element of $Z_{20} \oplus Z_{30}$ is 60.

2. (a) Define inner automorphism induced by an element 'a' of group G and find the group of all inner automorphisms of D_4.

(b) Defile the characteristic and the commutator subgroup of a group. Prove that the centre of a group is characteristic subgroup of the group.

(c) Let G' be the subgroup of commutators of a group G. Prove that G/G' is abelian. Also, prove that if G/N is abelian, then $N \geq G'$.

3. (a) Determine the number of cyclic subgroups of order 15 in $Z_{90} \oplus Z_{36}$.

(b) Define the internal direct product of the subgroups H and K of a group G. Prove that every group of order p^2, where p is a prime, is isomorphic to Zp^2 or $Zp \oplus Zp$ (external direct product of Zp with itself).

(c) Consider the group G = {1, 9, 16, 22, 29, 53, 74, 79, 81} under multiplication modulo 91. Determine the isomorphism class of G.

4. (a) Show that the additive group Z acts on itself by z.a = z+a for all z, a ∈ Z.

(b) Show that an action is faithful if and only if its kernel is the identity subgroup.

(c) Let G be a group. Let H be a subgroup of G. Let G act by left multiplication on the set A of all left cosets of H in G. Let π_H be the permutation representation of G associated with this action. Prove tha,t

(i) G acts transitively on A

(ii) The stabilizer of the point $1H \in A$ is the subgroup H.

(iii) $\text{Ker } \pi_H = \cap_{x \in G} xHx^{-1}$

5. (a) Let G be a permutation group on a set A (G is subgroup of S_A), let $\sigma \in G$ and let $a \in A$. Prove that $\sigma G_a \sigma^{-1} = G_{\sigma(a)}$, here G_x denotes stabilizer of x. Deduce that if G acts transitively on A then $\cap_{x \in G} \sigma G_a \sigma^{-1} = 1$.

(b) Show that every group of order 56 has a proper nontrivial normal subgroup.

(c) State Index theorem and prove that a group of order 80 is not simple.

6. (a) State the Class Equation for a finite group G, and use it to prove that p-groups have non trivial centers.

(b) Prove that group of order 255 is always cyclic.

(c) Show that the alternating group As does not contain a subgroup of order 30, 20, or 15.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1133 **C**

Unique Paper Code : 32357502

Name of the Paper : DSE-1 Mathematical Modelling and Graph Theory

Name of the Course : **B.Sc. (H) Mathematics – CBCS (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) (i) Determine whether x = 0 is an ordinary point, a regular singular point or an irregular singular point of the differential equation

$$xy'' + x^3y' + (e^x - 1)y = 0. \qquad (6)$$

(ii) Find the Laplace transform of the function $f(t) = 1 + \cosh 5t$.

(iii) Find the inverse Laplace transform of the function $F(s) = \frac{1}{s+5}$.

(b) Use Laplace transforms to solve the initial value problem :
$x'' - x' - 2x = 0;\ x(0) = 0,\ x'(0) = 2$ (6)

(c) Find two linearly independent Frobenius series solutions of
$2xy'' - y' - y = 0$ (6)

(d) Find general solutions in powers of x of the differential equation. State the recurrence relation and the guaranteed radius of convergence.
$5y'' - 2xy' + 10y = 0$ (6)

2. (a) Using Monte Carlo simulation write an algorithm to compute volume of the surface $x^2 + y^2 + z^2 \le 1$ that lies in the first octant $x > 0,\ y > 0,\ z > 0$. (6)

(b) Use Simplex method to solve the given linear programming problem (6)

Maximize : $3x_1 + x_2$
subject to $2x_1 + x_2 \le 6$
$x_1 + 3x_2 \le 9$
$x_1, x_2 \ge 0.$

(c) Consider a small harbor with unloading facilities for ships, where only one ship can be unloaded at

any time. The unicading time required for a ship depends on the type and the amount of cargo. Below is given a situation with 5 ships:

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships	10	20	45	50	75
Unload time	70	35	40	80	90

Draw the timeline diagram depicting clearly the situation for each ship. Also determine length of longest queue and total time in which docking facilities are idle. (6)

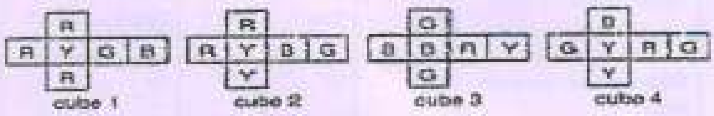
(d) Use Linear Congruence method to generate 15 random real numbers with multiplier 2, increment 5, modulus 13 and seed 1. Is there cycling? If yes, then give the period of cycling. (6)

3. (a) (i) Determine the number of edges of C_{10}, Q_5 and $K_{9,10}$. (3)

(ii) State and prove Handshaking Lemma. (3)

(b) Prove that a bipartite graph with odd number of vertices is not Hamiltonian. (6)

(c) Determine whether the given four cubes having four colours, can be stacked in a manner so that each side of the stack formed will have all the four colours exactly once. (6)



(d) By finding an Eulerian trail in K_5, arrange a set of fifteen dominoes [0 – 0 to 4 – 4] in a ring. (6)

4. (a) Use the factorization :

$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$

and apply inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{s^3}{s^4 + 4a^4}\right\} = \cosh at \cos at \quad (7)$$

(b) Fit the model to the data using Chebyshev's criterion to minimize the largest deviation, given the model y = cx and data set below : (7)

y	1	2	3
x	2	5	8

(c) Solve the initial value problem using the Laplace transform

$x'' + 4x' + 13x = te^{-t};\ x(0) = 0,\ x'(0) = 0$ (7)

(d) Name the five Platonic graphs. What is the degree of each vertex in each of these five graphs? Draw any two platonic graphs. (7)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1013 **C**

Unique Paper Code : 32351501

Name of the Paper : BMATH511 – Metric Spaces

Name of the Course : **B.Sc. (Hons.) Mathematics CBCS (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Let (X, d) be a metric space. Define the mapping $d^*: X \times X \to \mathbb{R}$ by

$$d^*(x,y) = \frac{d(x,y)}{1+d(x,y)};\ \forall x, y \in X.$$

P.T.O.

Show that (X, d^*) is a metric space and $d^*(x, y) < 1$, for every $x, y \in X$. (6)

(b) Let $\langle x_n \rangle_{n \geq 1}$ be a sequence of real numbers defined by $x_1 = a$, $x_2 = b$ and $x_{n+2} = \frac{1}{2}(x_{n+1} + x_n)$ for $n = 1, 2, \cdots$. Prove that $\langle x_n \rangle_{n \geq 1}$ is a Cauchy sequence in $\mathbb{R}$ with usual metric. (6)

(c) Define a complete metric space. Is the metric space $(\mathbb{Z}, d)$ of integers, with usual metric d, a complete metric space? Justify. (6)

2. (a) (i) Let (X, d) be a metric space. Show that for every pair of distinct points x and y of X, there exist disjoint open sets U and V such that $x \in U$, $y \in V$. (2)

(ii) Give an exam[illegible] of [illegible]

(a) A set in a metric space which is neither a closed ball nor an open set. (1)

(b) A metric space in which the interior of the intersection of an arbitrary family of the subsets may not be equal to the intersection of the interiors of the members of the family. (2)

(c) A metric space in which every singleton is an open set. (1)

(b) Let (X, d) be a metric space. Let A be a subset of X. Define closure of A and show that it is the smallest closed superset of A. (6)

(c) Let (X, d) be a complete metric space. Let $\langle F_n \rangle$ be a nested sequence of non-empty closed subsets of X such that $d(F_n) \to 0$. Show that $\bigcap_{n=1}^{\infty} F_n$ is a singleton. Does it hold if (X, d) is incomplete? Justify. (6)

3. (a) Let (X, d_X) and (Y, d_Y) be metric spaces and $f: X \to Y$ be a function. Prove that f is continuous on X if and only if $f(\bar{A}) \subseteq \overline{f(A)}$ for all subsets A of X. (6)

(b) Let A and B be non-empty disjoint closed subsets of a metric space (X, d). Show that there is a continuous real valued function f on X such that $f(x) = 0, \forall x \in A$, $f(x) = 1, \forall x \in B$ and $0 \leq f(x) \leq 1, \forall x \in X$. Further show that there exist disjoint open subsets G, H of X such that $A \subseteq G$ and $B \subseteq H$. (6)

(c) Define a dense subset of a metric space (X, d). Let $A \subseteq X$. Show that A is dense in X if and only if A^C has empty interior. Give an example of a metric space that has only one dense subset. (6)

4. (a) Show that the metrics d_1, d_2 and d_∞ defined on $\mathbb{R}^n$ by

$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|,$$
$$d_2(x,y) = \left(\sum_{i=1}^{n}(x_i - y_i)^2\right)^{1/2} \text{ and}$$
$$d_\infty(x,y) = \max\{|x_i - y_i| : 1 \le i \le n\}$$

are equivalent where $x = (x_1, x_2, \cdots, x_n)$ and $y = (y_1, y_2, \cdots, y_n)$. (6.5)

(b) Show that the function $f: \mathbb{R} \to (-1, 1)$ defined by

$f(x) = \frac{x}{1+|x|}$ is a homeomorphism but not an isometry. (6.5)

P.T.O.

(c) (i) Let (X, d) be a complete metric space. Let $T: X \rightarrow X$ be a [illegible] $d(Tx, Ty) < d(x, y)$, $\forall\, x, y \in X$. Does T always have a fixed point? Justify. (4)

(ii) Let X be any non-empty set and $T: X \rightarrow X$ be a mapping such that T^n (where n is a natural number, $n > 1$) has a unique fixed point $x_0 \in X$. Show that x_0 is also a unique fixed point of T. (2.5)

5. (a) Let ($\mathbb{R}$, d) be the space of real numbers with usual metric. Prove that a connected subset of E must be an interval. Give an example of two connected subsets of E, such that their union is disconnected. (4+2.5)

(b) Let (X, d) be a metric space such that every two points of X are contained in some connected subset of X. Show that (X, d) is connected. (6.5)

(c) Let (X, d) be a metric space. Then prove that (X, d) is disconnected if and only if there exists a continuous mapping of (X, d) onto the discrete two element space (X_0, d_0). (6.5)

6. (a) Prove that homeomorphism preserves compactness. Hence or otherwise show that

$$S(0,1) = \{z \in \mathbb{C} : |z| < 1\} \text{ and}$$

$$S[0,1] = \{z \in \mathbb{C} : |z| < 1\}$$

are not homeomorphic. (4+2.5)

(b) Let (X, d) be a metric space and $A \subseteq X$ such that every sequence in A has a subsequence converging in A. Show that for any $B \subseteq X$, there is a point $p \in A$ such that $d(p,B) = d(A,B)$. If B be a closed subset of X such that $A \cap B = \phi$, show that $d(A,B) > 0$. (4.5+2)

(c) Let f be a continuous real-valued function on a compact metric space (X, d_X), then show that f is bounded and attains its bounds. Does the result hold when X is not compact? Justify. (4+2.5)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper	**: 1132**	**C**
Unique Paper Code	: 32357501	
Name of the Paper	: DSE-1 Numerical Analysis (LOCF)	
Name of the Course	: B.Sc. (Hons.) Mathematics	
Semester	: V	

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All **six** questions are compulsory.

3. Attempt any two parts from each question.

4. Use of non-programmable scientific calculator is allowed.

P.T.O.

1. (a) Define fixed point of a function and construct an algorithm to implement the fixed point iteration scheme to find a fixed point of a function. Find the fixed point of $f(x) = 2x(1 - x)$. (6)

(b) Perform four iterations of Newton's Raphson method to find the positive square root of 18. Take initial approximation $x_0=4$. (6)

(c) Find the root of the equation $x^3 - 2x - 6 = 0$ in the interval (2, 3) by the method of false position. Perform three iterations. (6)

2. (a) Define the order of convergence of an iterative method for finding an approximation to the root of $g(x) = 0$. Find the order of convergence of Newton's iterative formula. (6.5)

(b) Find a root of the equation $x^3 - 4x - 8 = 0$ in the interval (2, 3) using the Bisection method till fourth iteration. (6.5)

(c) Perform three iterations of secant method to determine the location of the approximate root of the equation $x^3 + x^2 - 3x - 3 = 0$ on the interval (1, 2). Given the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the approximations just obtained. (6.5)

3. (a) Using scaled partial pivoting during the factor step, find matrices L, U and P such that LU = PA

where $A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{pmatrix}$ (6.5)

(b) Set up the SOR method with w=0.7 to solve the system of equations:

$$3x_1 - x_2 + x_3 = 4$$

$$2x_1 - 6x_2 + 3x_3 = -13$$

$$-9x_1 + 7x_2 - 20x_3 = 7$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Jacobi iteration scheme to solve the system of equations:

$$10x_1 + x_2 + 4x_3 = 31$$

$$x_1 + 10x_2 - 5x_3 = -23$$

$$3x_1 - 2x_2 + 10x_3 = 38$$

Take the initial approximation as $X^{(0)} = (1, 1, 0)$ and do three iterations. (6.5)

4. (a) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data:

x	1	2	4	8
f(x)	3	7	21	73

(6)

(b) Calculate the Newton second order divided difference $\frac{1}{x^2}$ of based on the points x_0, x_1, x_2. (6)

(c) Obtain the Lagrange form of the interpolating polynomial for the following data:

x	1	2	5
f(x)	-11	-23	1

(6)

5. (a) Find the highest degree of the polynomial for which the second order backward difference approximation for the first derivative

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}$$

provides the exact value of the derivative irrespective of h. (6)

(b) Derive second-order forward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)}{2h}.$$

(6)

(c) Approximate the derivative of $f(x) = \sin x$ at $x_0 = \pi$ using the second order central difference formula taking $h = ½$, ¼ and 1/8 and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the Simpson's rule, approximate the value of the integral $\int_2^5 \ln x \, dx$.. Verify that the theoretical error bound holds. (6.5)

(b) Apply Euler's method to approximate the solution of initial value problem $\frac{dx}{dt} = \frac{e^t}{x}, 0 \le t \le 2,$, $x(0) = 1$ and $N = 4$.

Given that the exact solution is $x(t) = \sqrt{2e^t - 1}$,, compute the absolute error at each step. (6.5)

(c) Apply the optimal RK2 method to approximate the solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $1 \leq t \leq 2$, $x(1) = 1$ taking the step size as $h = 0.5$. (6.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1233 **C**

Unique Paper Code : 32357507

Name of the Paper : DSE – 2 Probability Theory and Statistics

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions selecting any **two** parts from each questions no.'s **1** to **6**.
3. Use of scientific calculator is permitted.

P.T.O.

1. (i) If the random variable be the time in seconds between incoming telephone calls at a busy switchboard. Suppose that a reasonable probability model for X is given by the probability density function :

$$f_X(x)=\begin{cases}\frac{1}{4}e^{-\frac{x}{4}} & 0<x<\infty\\ 0 & \text{elsewhere}\end{cases}$$

Show that f_X satisfies the properties of a probability density function. Also show that the probability that the time between successive phone call exceed 4 seconds is 0.3679. (6)

(ii) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2)=\begin{cases}8x_1x_2 & \text{if } 0<x_1<x_2<1\\ 0, & \text{elsewhere}\end{cases}$$

Find $E(X_1X_2^2)$, $E(X_2)$, $E(7X_1X_2^2+5X_2)$. (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y)=\begin{cases}e^{-y} & \text{if } 0<x<y<\infty\\ 0, & \text{otherwise}\end{cases}$$

Find the marginal pdf of X and Y. (6)

2. (i) Let X be a continuous random variable with pdf

$$f(x)=\begin{cases}kxe^{-\lambda x}, & x\geq 0,\ \lambda>0\\ 0, & \text{otherwise}\end{cases}$$

Determine the constant k, mean, variance and the cumulative distribution function of X. (6)

(ii) If a random variable X is uniformly distributed over the interval [α, β] then find the mean, variance and moment generating function of X. (6)

P.T.O.

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} 6y & \text{if } 0 < y < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the E(Y|x) and E[E(F|x)]. (6)

3. (i) Let (X, y) be a random vector such that the variance off is finite. Then show that Var[E(Y|X)] ≤ Var(Y). (6)

(ii) If X is a binomial variate with parameter n and p then prove that

$$\mu'_{r+1} = \left[np\mu'_r + pq\frac{d\mu'_r}{dp}\right], \text{ where } \mu'_r = E\left[x^r\right] \text{ and r}$$

is a non-negative integer. (6)

(iii) Let the random variables X and Y have the linear conditional means E(Y|x) = 4x + 3 and $E(X|y) = \frac{1}{16}y - 3$. Find the mean of X, mean of Y, the correlation coefficient. (6)

4. (i) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & \text{if } 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Show that X_1 and X_2 are not independent. (6.5)

(ii) State and prove the Chebyshev's Theorem. (6.5)

(iii) If the probability is 0.25 that an applicant for driver's license will pass the road test on the given try, what is the probability that an applicant will finally pass the test on the fourth try? (6.5)

5. (i) Calculate the correlation coefficient for the following age (in years) of husband's (X) and wife's (Y) : (6.5)

X	23	27	28	28	29	30	31	33	35	36
Y	18	20	22	27	21	29	27	29	28	29

(ii) If X and Y have a bivariate normal distribution, the conditional density of Y given X = x is a normal distribution with the mean,

$$\mu_{Y|x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1)$$

and the variance

$$\sigma^2_{Y|x} = \sigma^2_2\left(1-\rho^2\right)$$ (6.5)

(iii) The joint density of X_1, X_2 and X_3 is given by

$$f(x_1, x_2, x_3) = \begin{cases}(x_1 + x_2)e^{-x_3} & \text{if } 0 < x_1 < 1,\ 0 < x_2 < 1,\ 0 < x_3 \\ 0, & \text{elsewhere}\end{cases}$$

Find the regression equation of X_2 on X_1 and X_3. (6.5)

6. (i) Two fair dice are tossed 600 times. Let X denote the number of times a total of 7 occurs. Use Central limit theorem to find $P[95 \le X \le 115]$. (6.5)

(ii) To show how an exponential distribution might arise in practice. If random variable X has an exponential distribution with parameter θ then find its mean, variance and moment generating function. If X has exponential distribution with mean 2 then find $P[X < 1]$. (6.5)

(iii) If X is a random variable having a binomial distribution with parameter n and θ, then the moment generating function of $Z = \frac{X - n\theta}{\sqrt{n\theta(1-\theta)}}$

approaches that of the standard normal distribution when $n \to \infty$. (6.5)